



## RESOLVING DIFFICULTIES IN LEARNING MATHEMATICS USING CONCEPTUAL CHANGE APPROACH

R.Prabhu<sup>1</sup> | Dr.N.Muthaiah<sup>2</sup>

<sup>1</sup> M.Sc., M.Ed., M.Phil(Maths), M.Phil(Edn), M.Phil(Comp.Sci) SET., Assistant Professor & Ph.D Research Scholar, Sri Ramakrishna Mission Vidyalaya College of Education (Autonomous), Coimbatore – 641 020.

<sup>2</sup> Ph.D., Principal, Sri Ramakrishna Mission Vidyalaya College of Education (Autonomous), Coimbatore – 641 020.

### ABSTRACT

*This article deals with misconceptions that exist in learning mathematics. By virtue of development of conceptual change approach, the so called "Difficulties" in solving problems in mathematics would be resolved and students get "Pleasure" in solving problems. In this study, addition of numbers, basic operations in set language and decimal number system are taken for consideration and the conceptual change approach has been discussed.*

**Keywords:** Conceptual change approach, mathematics, procedural knowledge, decimals.

### Introduction

All human cognition begins with intuitions, proceeds from thence to conceptions and ends with ideas. Errors are very interesting and revealing, not just of a student's current state of knowledge or understanding, but of the process of thinking itself, and strategies fortackling difficult or sophisticated mathematical problems. There are at least two types of mathematical reasoning that tend to lie at the opposite ends of a spectrum. They may, if one is aware of them, interact, balance and reinforce each other, to further learning and deepen an appreciation of the particular phenomenon under investigation. On the one hand, we have syntactic reasoning, which, broadly speaking, one associates with the superficial end of the learning spectrum, and relies on simple, incremental rules, searching or pattern matching. It can, for example, involve very literal interpretations and superficial relationships, pasting together ideas, almost like a collage. It is extremely common, especially when students are under pressure in examinations, or with a cascade of deadlines, and must respond quickly without thinking too carefully about their answers. Semantic reasoning, on the other hand, is more naturally associated with the deeper end of the learning spectrum, and relies on solid intuition, insight or experience.

This dichotomy between syntax (form) and semantics (meaning) is ancient and goes back to Euclid's Elements, the first published attempt to create an axiomatic deductive system in mathematics, providing a paradigm for the formal development of any area of mathematics. This led to the question whether mathematics could, in some precise sense, be reduced to syntax through the formal and mechanical manipulation of axioms.

### Conceptual and procedural knowledge in Mathematics education

Computation is an important part of mathematics and

proficiency in computation has long been one of the main objectives of mathematics teaching and learning in school and college level. However, the existence and efficiency of computational tools, such as calculators and computers, prove that parts of what is included in the common notion of procedural knowledge can be achieved without understanding. It is also often witnessed that students who, for example, solve an equation correctly, sometimes do not know what to do if the task is to answer if a given number is a solution to the same equation. Such behaviour indicates a poor conceptual understanding concerning equations.

Mathematical techniques to solve certain types of problems can thus be learnt without referring to a discursive level of justification why the techniques work or how the mathematical notions used are to be understood and can be related to other concepts.

Hence students should be made to understand that 'problem solving' not only involves in getting the desired result but also to understand the basic concept involved in it. The more, the students learn the concepts fully, the result of obtaining answers would be simple and interesting.

For example, when students of elementary level are asked to find the value of  $4 \times 4$ , they may obtain the result by rote learning of multiplication tables. But teachers should explain the methodology of multiplication and the inter relationship of addition with multiplication. Initially, students may be asked to physically count, take for example, the rows and columns of beads. Thus students count the number of beads placed row & column wise to find the solution. Thus the concept of multiplication would be very easily understood by the students without any confusion. For this, all out efforts should be taken for the same thus resulting in easy mathematics. This will not only yield very good results in solving mathematical problems but also makes the mind induced to learn effectively.

### ***Conception with respect to content knowledge***

Basically concept development is the core of any content for any level of education. Thus before understanding the content on any subject the concept is to be understood which results in the overall success level of content developed. While framing content, it should be kept in mind what are all the concepts that could be directly or indirectly taught to the students by virtue of introduction of those contents.

According to Shulman (1986), teacher content knowledge – subject matter content knowledge, pedagogical content knowledge and curricular knowledge – are intertwined in practice. Pedagogical content knowledge includes “*an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them ... If those preconceptions are misconceptions, which they often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners.* (Shulman, 1986, pp. 9-10)

Subject matter knowledge is more than knowledge of facts or concepts – it requires knowledge of both the substantive structure (facts and their organising principles) and syntactic structure (legitimacy principles for the rules) of a subject domain. The transformation of subject matter knowledge into pedagogical content knowledge is a significant focus in teacher education (Goulding, Rowland & Barber, 2002). We suggest that pre-service teachers who confront their own mathematical errors, misconceptions and strategies in order to reorganise their subject matter knowledge, have an opportunity to develop a rich pedagogical content knowledge.

### ***Resolving difficulties in addition of whole numbers at elementary stage using conceptual change approach***

Before addition can be defined, certain terms must be defined. Addition is a binary operation. That is, the operation is always performed on two numbers. Note that to add  $5 + 7 + 3$  it is necessary to first perform addition on two numbers and then another addition on two numbers. Thus,  $5 + 7 + 3$  becomes  $(5+7)+3 = 15$ . To be an operation, a process must be performable and the result must be unique. The numbers to be added ( $5+7$ ) are called addends. The result of the operation is called the sum ( $5+7=12$ ).

These problems may be practically displayed with physical objects so that the children understand the concept of addition without any problem. For example, in order to avoid a common difficulty, the concept of the operation “union of sets” should not be considered the same as “addition of numbers”. Children have often been told (particularly in algebraic concepts like variables and constants), “You cannot add unlike things”, One does not add things, but numbers.

It is possible to form a union of a set containing three

apples and a set containing six mangoes. The set formed is a set of fruit containing three apples and six mangoes. We may also think of the union of three apples and six mangoes as a set containing nine pieces of fruit.

It may also be explained to make children understand the concept without any ambiguity with the following simple example.

Three sticks of 1 foot long are not the same thing as one stick 3 feet long, although  $1 + 1 + 1 = 3$ . They are equivalent in length, however.

While the addition of numbers is “finding the sum without counting”, the basis of addition of whole numbers is founded upon counting. The beginning of the addition idea starts in the earliest days of school life when pupils combine groups of objects and count to find “how many?” Soon many children recognize without counting that the result of combining a set of 3 objects with a set of 2 objects is a set of 5 objects. At this time, they are adding.

It is often suggested that pupils should follow a three step program in developing mathematical abstractions” first, a *concrete stage* in which pupils work only with the actual objects; second, a *semi-concrete* stage in which counters and pictures are used to replace the actual objects; third, the *abstract stage* in which numerals are used to represent numbers. In actual practice, children often do not need all three stages. If orally presented verbal problems are used, some pupils can think from the concrete problem to semi-concrete material or to the actual number idea. Because children vary greatly in their maturity in abstract thinking, opportunities should be provided for the pupil to choose appropriate methods of understanding early addition situations.

### ***Resolving difficulties in Decimals at elementary stage using conceptual change approach***

A number such as 4.56 may be read in several ways – four and fifty six hundredths, four point fifty-six, or four point five six. The use of the word “point” in the reading of a decimal has the distinct advantage of indicating the suggested decimal notation. For example, if someone reads, “seven and three tenths,” and asks for the numeral indicating this to be written, the listener does not know

whether  $7\frac{3}{10}$  or 7.3 is wanted. “Seven point three”

indicates that the decimal form is preferred for this case.

The use of the “and” should be reserved for decimals. In common usage, 149 is often read as one hundred and forty nine. In this case, no error will be made. However, it is difficult to ascertain the meaning of two hundred and twenty eight thousandths. If the “and” is used with whole numbers as well as fractions, the pupil does not know whether the meaning is 200.028 or 0.228.

A rational number may be named by either a “fraction” or a “decimal. The term “decimal” rather than “decimal fraction” is preferred because  $3/10$  can be considered a decimal fraction. Thus  $3/10$  is called a fraction, and 0.3 a

decimal. It should be noted that there is no complete agreement among writers in mathematics education. Some will refer to decimal fractions.

Decimals and fractions are two types of symbolism for numbers (rational numbers). Fractions have numerators and denominators; with decimals, the denominator is not written. A number may be named either a "fraction" or a "decimal". The terms "fractions" and "decimals" for the names of numbers are preferable to the older terms of "common or vulgar fractions" and "decimal fractions." The restrictive use of the word "decimals" rather than "decimal fractions" also has the advantage of clearing up questions such as "Is  $3/10$  a decimal fraction?"

It is often helpful to place a zero before the decimal point for decimals with a value less than one for example, 0.234.

Emphasis should be placed upon the *ones* place as central to decimal notation. This usage emphasizes structure, balance, and symmetry. Too often the decimal point is taught as the center of the system.

#### Correct Method

Thousands   Hundreds   Tens   Ones   .   Tenths   Hundredths   Thousandths

#### Incorrect Method

Thousands   Hundreds   Tens   Ones   .   Tenths   Hundredths   Thousandths

The steps in reading a decimal are i) read the part of the numeral to the left of the decimal point, ii) read the decimal point as "and" or "point," and iii) read the entire part of the numeral to the right of the decimal point as though it represented a whole number, following the statement with the place value of the digit on the far right.

### **Conclusion**

Before solving any problem in mathematics, the concept of the problem should be unambiguously understood. Thus, in order to assess the learning ability of children, the myths and misconception of mathematical concepts are to be ascertained.

It is concluded from the study that Misconceptions in Mathematics can be resolved by making children understand the basic elements and then the core of the problems. For this, the teachers have to select the proper method of teaching based on the perception level of the students. As a result, the students attentively involve in the process of learning without any mental fatigue and could interestingly understand the original concept of mathematical topics thus resulting success on the content framed.

### **REFERENCES**

1. Alan Riedesel C, (1967), "Guiding Discovery in

*Elementary School Mathematics*", Meredith Publishing Company, New York.

2. Johann Engelbrechta\*, Christer Bergstenb and Owe Ka° gesten, (2009) "Undergraduate students' preference for procedural to conceptual solutions to mathematical problems", University of Pretoria, South Africa; Linköping University, Sweden

3. Julie Ryan, Liverpool John Moores University & Barry McCrae, Australian Council for Educational Research.

4. Gardinar, A. (1987), "Mathematical Puzzling", Dover Publications Inc., Mineola, New York.

5. Goulding, M., Rowland, T., and Barber, P. (2002) *Does it matter? Primary teacher trainees' subject knowledge in mathematics*, British Educational Research Journal, 28(5)

6. Hardy G.H., (2005), "A Mathematician's Apology", University of Alberta Mathematics, Simon Society, Canada.

7. Kapur, J.N., (1981), "The Spirit of Mathematics", Arya Book Depot, New Delhi.

8. Lee E. Yunker and Joe Cross White (1984), "Advanced Algebra", Merril Publishing Company, Columbus, Ohio.

9. Polya. G., (2002), "How to Solve It", Princeton University Press, Princeton.

10. Shulman. L. S., (1986) "Those Who Understand: Knowledge Growth in Teaching", Educational Researcher, Vol. 15, No. 2 (Feb., 1986), pp. 4-14

11. Shulman, L. S. (1987). *Knowledge and teaching: Foundations of the new reform*. Harvard Educational Review, 57(1), 1-22.

### **Web References :**

1. <http://www.pleacher.com/handley/poetry/mpoet.htm>
2. <http://www.mth.uct.ac.za/digest/twoplustwo.html>
3. <http://www.goodreads.com/quotes/tag/math>
4. <http://www.mathstory.com/>
5. <http://www.tooter4kids.com/>