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HOW TO TEACH THE COMBINATORY PART OF DIGITAL ELECTRONICS BASIS WITH PROJECT PEDAGOGY ? THANKS TO A SELF-WORKING CARD TRICK NAMED "CYCLIC NUMBER" !

*Pierre SCHOTT

Art et Recherché NUMérique (ARNUM), Ecole Supérieure d'Informatique, d'Electronique et d'Atomatisme (ESIEA), Paris, France - 75005.

ABSTRACT

Why use Magic for teaching digital electronics theory and software? Magicians know that, once the surprise has worn off, the audience will seek to understand how the trick works. The aim of every teacher is to interest their students, and a magic trick will lead them to ask how? And why? And how can I create one myself?
Whatever the student's professional ambitions, they will be able to see the impact that originality and creativity have when combined with an interest in one's work.
The students know how to "perform" a magic trick for their family and friends, a trick that they will be able to explain and so enjoy a certain amount of success. Sharing a mathematical / informatics demonstration not easy and that they do so means that they will have worked on understood and are capable of explaining this knowledge. Isn't this the aim of all teaching?

In this article I present a self-working magic card trick. Using this card trick to teach a full combinatorial and sequential digital electronic course is actually possible using project pedagogy or classical top-down method. I present the global study in order to use a project pedagogy, then a part of the synthesis.
In fact, this trick can be used to teach: Binary number system and weighted codes, Transcoders : binary to weighted code, logic minimization using Karnaugh, seven segment displays with the associated encoders and decoders, multiplexers and demultiplexers, an open source software named "Logisim".

KEYWORDS: Higher education, engineer, educational method, digital electronics software, combinatory circuits, project pedagogy.

1 Introduction

I am fascinated by Magic (or rather conjuring!) and for many years I have used this way of teaching, both in my physics classes and as higher education teacher trainer.

I present in this paper firstly all the combinatory digital electronics notions I can teach with the cyclic number, secondly a brief history of Magic and finally the researchers who have used this Art to teach and/or to research.

The aim of the magician is to hide the principles he uses (using Maths, Physics, Psychology, sleight of hand, etc...) by disguising the trick so that the audience has no way of discovering how it is done thus allowing the Magic to remain.

The teacher can do exactly the opposite: unraveling a Magic trick to highlight the principles used!

1.1 Summary

Through this paper, we will make students gather information about all these following notions at the time they need to, in order to keep going forward in their project:

- Binary number system and weighted codes;
- Transcoders : binary <-> weighted code;
- Logic minimization using Karnaugh map (for example to synthesize these transcoders using the gates AND, OR, NOT);
- Synthesize comparators thanks to other gates (NOT, NOR) and/or by arithmetic circuits;
- Seven segment displays with the associated encoders and decoders;
- Multiplexers and demultiplexers;
- An open source software named "Logisim";
- D-Latch, Flip-Flop Gate (in part II);
- Synchronous and asynchronous counters and circuits (in part II).

Obviously, students will want quickly to use a simulator... and I hope so! That will get them to learn a new simulation software as well as learning basic notions.

1.2 A brief history of Magic

From the beginning of time people have feared what they don't understand and sought logical explanations for inexplicable phenomena. To begin with they considered them to be the work of magic, then the work of the gods, then the work of God himself. The church discouraged the spread of the conjurer's art as it preferred not to have rational explanations for what was considered supernatural.

The first magic tricks were performed in the Middle Ages by clowns and/or conmen who would entertain passers-by by getting them to bet on the position of a ball hidden in one of three up-turned goblets, the bet being usually lost. This trick is known nowadays by the name 'cups and balls' (Mayol, 2000). The first book considered to be about modern Magic (or should we say "conjuring") was written and printed in the 16th Century. It was about Magic with ropes (Ammar, 1998). It wasn't until the end of the 19th Century that the word Magic took on its present-day meaning when the famous magician Houdini, father of modern Magic, made it the art it is today.

Since then a good number of principles have been invented and improved on by magicians and gamblers (Erdnase, 1902), especially for card tricks with bets. Since the 1980s the secrets that were once passed on from master to apprentice are now universally available through the use of video cassettes and modern communication technology, and Magic has become Big Business.

1.3 Card magic as the vector of research and teaching

From 1886 till 1896, Poincaré occupied the chair of "probability Calculus" in the Paris university 'La Sorbonne'. He wrote a work named 'probability calculus' (Poincare, 1912 ; Sheynin, 1991) which was printed for the first time in 1896. In the second edition, he brings very fundamental new reflections on the groups and the hypercomplex systems and on the ergodic theory. He is brought to these innovations by the study of the card shuffling and liquids mixing. The problems of card shuffling and liquid diffusion studied by Poincaré are application cases of the ergodic theory which is in the center of the probability leveling phenomenon: if the deck was shuffled for a long time, all the possible permutations have the same probability.

Three principles of Gilbreath (Gilbreath, 1958 ; Gilbreath, 1966 ; Gilbreath, 1989) are fascinating principles, allowing to do extraordinary card tricks! With A.Lachal, we have given the mathematical demonstrations of all the principles (Lachal, 2012 ; Lachal, 2013)

Some Mathematicians -– such as M.misters Gardner (Magid, 2005 ; Gardner, 1958 ; Gardner, 2005), P.Diaconis (Diaconis, 1998 ; Diaconis, 2003 ; Assaf, 2009) or C.Mulcahy (Mulcahy, 2003 ; Mulcahy, 2004 ; Mulcahy, 2007)-, some computer specialists - as G. Huet (Huet, 1991) one of creator of COQ language which allows to make automatic proof mathematical proof - bent overstudied this principle (and they are not the only ones !).

We shall not present this principle here but it is necessary to know that it is based on a very commonly used cards shuffles besides on both sides of the Atlantic Ocean: the American shuffle. With this only shuffle, informatics, mathematics, probability notions can be teached! (Schott, 2011)

1.4 The Hhigher education in FRANCE
In France, the secondary education is sanctioned by the high school diploma. Ideally the pupils obtain it at the age of 18. Then, they have a plethora of possibilities following 3 axesorientations:

- Some undertake short studies called "professional" in an I.U.T at the University or in high school with the aim of obtaining a BTS. Both programs lead to a 2-year technical degree,

- Others undertake longer studies. In this case, if they have obtained their high school diploma they can:

- Enrol in university,

- Or enter preparatory courses (classes for entrance into 'Grandes Ecoles') for 2 years, which will lead them, via national entrance examinations, to enter a 'Grande Ecole' in business or engineering.

The studies take 3 years and once they have their degree in hand, the students find employment easily.

The school in which I work, named E.S.I.E.A, is an engineering school.

1.5 My motivations to propose such kind of course
I teached since many years with a descending method the introduction of digital electronics lecture. But the subject of the final work, I gave to the students, was always a peculiar subject such as a music partition, fitness program or the cyclic number card trick.

I thought I can build my lecture on this subject with an ascending method like the pedagogy project!

I chose the topic of magic, a subject which fascinates me, to be virtually certain that the topic would be completely new for the students. So they are in an unknown unfamiliar situation. They will thus have to implement all the necessary means to complete the project successfully.

I also try to arouse the students' curiosity by proposing a subject that is more playful than usual.

Following these objectives, I propose a self-working magic card trick. The aim of the course/project is to synthetize this card trick with an open source digital electronics simulator.

Consequently, the students have to look for, find and understand a 'new' theory of digital electronics and new software: Logisim.

Finally, I try to change the traditional way of teaching: a downward method (the professor teaches the student) usually used in France, become an ascending method here (it is the student who will have to explain to the professor).

The ascending method has the merit of showing whether the posed problem posed is well understood. As Boileau wrote (Boileau, 1740)Boileau " An idea well conceived presents itself clearly, and the words to express it come readily" ("ce qui se conçoit bien s'énonce clairement, et les mots pour le dire arrivent aisément").

2 Materials and Methods: Using the self-working card trick based on the cyclic number 142857 (1/7=0.142857)

The magic arts consist of several specialties: close - up, cups and balls, coin tricks, transformists, card tricks, and so on ... The most mediatized specialty, enjoyed by all kinds of audience, is: the Grand Illusion. But the easiest is self-working card tricks.

In fact, many magic tricks are based on mathematical notions. The art of the magician consists in the fact that the spectators do not realize it!

"The number 142,857, which students of recreational number theory are likely to recognize at once, is one of the most remarkable of integers. Apart from the trivial case of 1, it is the smallest "cyclic number." A cyclic number is an integer of n digits with an unusual property: when multiplied by any number from 1 through n, the product contains the same n digits of the original number in the same cyclic order. Think of 142,857 as being joined end to end in a circular chain. The circle can be broken at six points and the chain can be opened to form six digit numbers, the six cyclic permutations of the original digits:

1 X 142857 = 142857
2 X 142857 = 285714
3 X 142857 = 428571
4 X 142857 = 571428
5 X 142857 = 714285
6 X 142857 = 857142

The cyclic nature of the six products has long intrigued magicians, and many clever mathematical prediction tricks are based on it (Regnault,1943) (Costa Aliago,1953) (Simon,1964). Here is one:" (Gardner, 1992).

To find this kind of number with the same properties, take the prime number, divide it by 1 and if the digit number of the perodic decimal part is equal to the number minus one, then you have found a new cyclic number! Here 1/7=0,142857 142587 …

2.1 The self-working card trick effect
"Prepare a deck of playing cards by removing the nine spades and nine hearts with digit values. Place them on the bottom of the deck so that their order from the bottom up is 1, 4, 2, 8, 5, 7, with the remaining three cards following in any order. Place away the 6 selected hearts as a prediction […] A random multiplier from 1 through 6 is now obtained by rolling a die. Better still, hand someone an imaginary die and ask him to roll this invisible die and tell you what he "sees" on top. Multiply 142,857 by the digit he names. Take and then cut the prediction deck (in order to determine where to cut, multiply 7 by the selected digit to get the last digit of the product)." (Gardner, 1992).

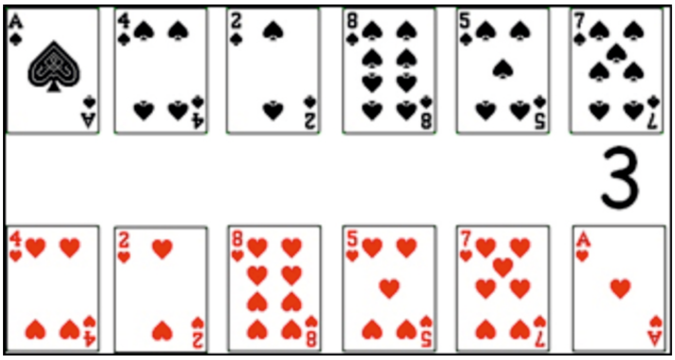

Figure 1. Introducing the 142857 trick with cards and a dice.

3 Results :
3.1 Global Study and how to cut in pedagogical sequences with ascending pedagogy
The first sequence! Firstly I perform the magic trick and then I propose the students to realize it … using digital electronics (with seven segments displays) … but the students have to figure it out! Secondly, and it's the aim of this first sequence, consists

in breaking up this large problem into smaller bits. It should achieve the anticipated outcome: to find smaller unity problems (but students have to think, they had "alone" find the solutions) and potentially in the opposite order shown in the "normal" top-down pedagogical method:

- Using a cycle (142857) with different connections to achieve the project;

- Several representations of a single number (coming from a card or the dice) are needed, hence the introduction of binary code and balanced code;

- How to give rhythm to the cycle? We introduce here flip flops as part of using up-counter and down-counter;

- I personally don't want to use any of these counters but a very special cycle so I have to synthetize control circuits. So I have to introduce combinatory logic;

- In order to create these control circuits, we have to make functions, hence the introduction of the SYSTEM's truth tables and the use of Karnaugh maps;

- But then we need to know some elementary gates to synthetize with gates the results of the Karnaugh maps approach.

- At the end, I have to display the numbers in order to perform the magic show. That's why, we have to study the seven segment displays with the associated encoders and decoders! But we have 13 displays and we have to select one of them thus the MUX and DEMUX circuits must be used.

So, all of the needed notions are found and the next step can be reached. Each week, the students have to find the theoretical notions for each single problem then find a solution and finally implement it. The teacher work consists "only" to be sure that the theoretical found notions by the students are right and that the students have well understood them! If necessary, the teacher can (must) expose and explain the notions with the found informations; that's very important that the teacher use ONLY the found notions... the figure 2 shows the complete digital circuit for the 142857 magic trick.

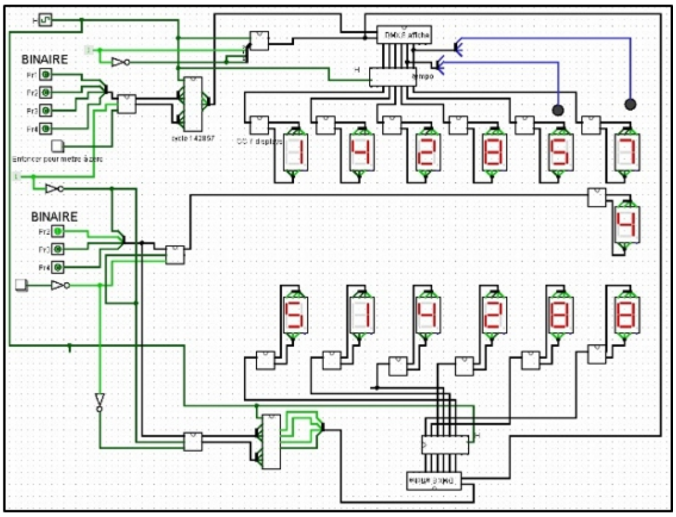

Figure 2. The complete 142857 digital electronics circuit.

3.1.1 Project representation by states

We first have to create the cycle that represents the 142857 number as shown on the figure 3:

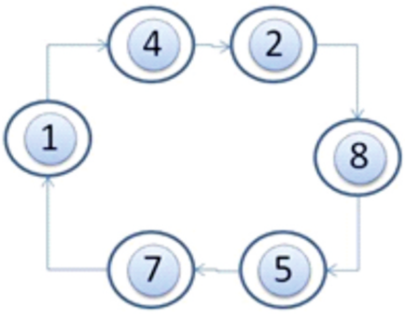

Figure 3. Cycle in decimal basis of the cyclic used number: 142857

Then, we have to draw the new entry point in the cycle from the second operand (the multiplicative number between 1 and 6). This is done and shown on this figure 4:

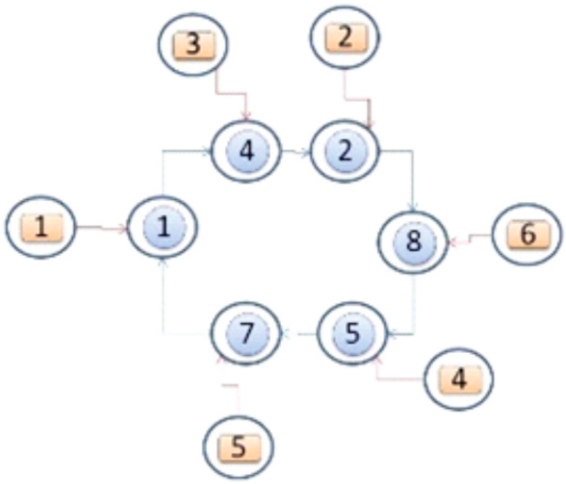

Figure 4. Complete cycle in decimal basis of the cyclic used number 142857

It easily comes readily that we have to code the same decimal number in two different ways: it's the entry point to introduce the binary coding.

We actually count how many states we have. Here we have 12 states and the aim is to find n so 2n-1 < 12 ≤ 2n. Thus n=4 and it's the number of flip-flop gates that we needed for a synchronous or asynchronous counter.

3.1.2 Binary code and binary weighted code

If we use in our problem to represent each state the binary representation it's impossible to determine if the 5 is a part of the cycle or it's the operand! That's why we have to used the weighted 1-4-2-1 code is used.

Table 1 shows the decimal equivalent of every possible combinations of a 4-bits binary number. Its equivalent in weighted 1-4-2-1 code is also given.

Table 1. Representation of decimal numbers into binary and weighted code systems 1-4-2-1.

decimal	Binary (b3b2b1b0)	Code 1-4-2-1	decimal	Binary (b3b2b1b0)	Code 1-4-2-1
0	0000	0000	8	1000	1111
1	0001	1000 0001	9	1001	X
2	0010	0010 1001	10	1010	X
3	0011	1010 0011	11	1011	X
4	0100	0100 1011	12	1100	X
5	0101	1100 0101	13	1101	X
6	0110	0110 1101	14	1110	X
7	0111	1110 0111	15	1111	X

For the decimal number which have 2 representations, a choice must be done. The first representation is taken to represent the number in the cycle and the second one for the operand. So, the complete project model is given on figure 5, using the figure 4.

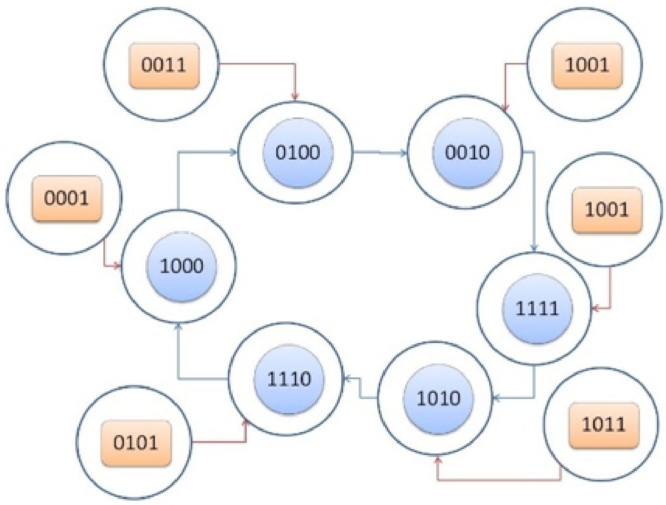

Figure 5. Entire cycle, with the weighted 1-4-2-1 code, of the number 142857

3.1.3 Conclusion of the study

We have to synthetize 4 macros circuits:

- a transcoder binary -> weighted code;
- a "transcoder" weighted code -> the display;
- a multiplexer to select a 7 segment displays;
- a synchronous or asynchronous counter with many command circuits before the each flip-flop gate of the counter.

3.2 One example of a combinatory circuit : binary to weighted code transcoder synthesis

The logical equations obtained either by the truth table or the Karnaugh maps must be required in order to synthetize and then construct all of the combinatory circuits. Find these equations for the chosen transcoder. All of the figures are obtained by Logisim.

3.2.1 Truth tables

Let say that the binary representation of a decimal number is given by 4 bits (b3b2b1b0). The weigthed code representation of the same decimal number is given by (cwb3cwb2cwb1cwb0).

The truth table for the cycle circuit is given on table 2 where the binary number is the entry of the circuit and the binary weighted number is the output.

Table 2. Representation of decimal numbers into binary and weighted code systems 1-4-2-1.

b3	b2	b1	b0	cwb3	cwb2	cwb1	cwb0
0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	1	0	0	0	1	0
0	0	1	1	1	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	1	1	0	0
0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	0
1	0	0	0	1	1	1	1
1	0	0	1	x	x	x	x
1	0	1	0	x	x	x	x
1	0	1	1	x	x	x	x
1	1	0	0	x	x	x	x
1	1	0	1	x	x	x	x
1	1	1	0	x	x	x	x
1	1	1	1	x	x	x	x

A minterm (resp. maxterm) is a Boolean expression resulting in 1 (resp. 0) for the output of a single cell, and 0s (resp. 1s) for all other cells in a Karnaugh map, or truth table. For example, the minterm of the cwb3 output is :

To reduce the number of gates, the Morgan's law can be applied. But the easiest way to do that is the use of Karnaugh map.

$$cwb_3=\overline{b_3}\,\overline{b_2}\,\overline{b_1}\,b_0+\overline{b_3}\,b_2\overline{b_1}\,b_0+\overline{b_3}\,b_2b_1b_0+b_3\overline{b_2}\,\overline{b_1}\,\overline{b_0} \quad (1)$$

Be careful: a second transcoder must be synthetized to pass from binary representation to the second representation of the weighted code to represent this time the operand! We need to introduce another input 'n' which indicates that if we want to convert the binary number using the first or second possibility to write the same decimal number

3.2.2 Karnaugh map

In order to simplify the truth table given on table 2, we are going to use Karnaugh map presented on figure 6.

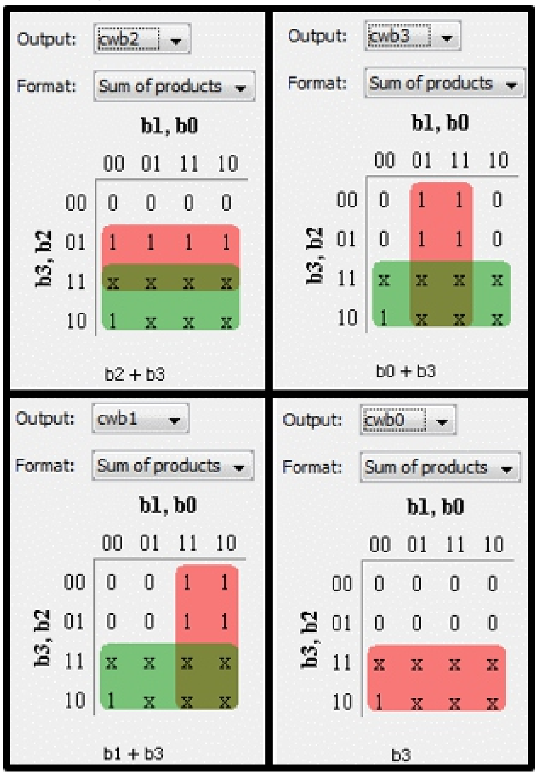

Figure 6. Karnaugh map in order to simplify the truth table of the transcoder binary -> weighted code 1-4-2-1

So, the equation (1) is simplified as follow: $cwb_3=b_0+b_3$

3.2.3 Synthesis of combinatorial circuits from logic equations

To synthesize these logic equations, let's introduce standard logic gates. Thus, the studied transcoder is synthesized by the logic circuit of the figure 7:

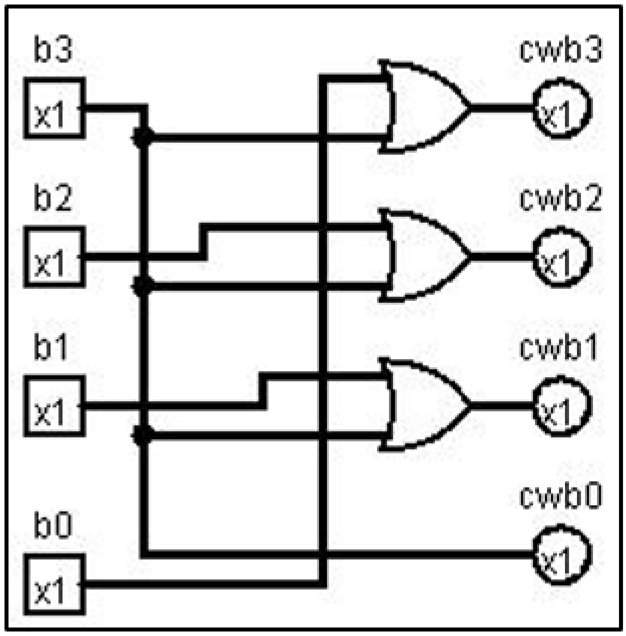

Figure 7. Electronic circuit of the first transcoder binary->weighted code 1-4-2-1

3.2.4 Tests

At each step of the synthesis, we have to test it! For that, we can use a 7 segment display but to use it, we have to synthesize an another transcoder!

He easiest way is to use the "Simulation" menu in Logisim. Ad test all of possibilities.

When the circuits is verified and tested, we can go further with the second part!

4 Discussion:

It's easily understandable to teach Math or Optics Illusion with magic. In this paper, I have proposed to teach combinatory part of the digital electronic using Magic and the experience shows that it works quiet good!

I think that it seems easy to tech as explained but if you have never used the down-top pedagogy, you have:

- to give a reading packet which the students must fill up. At the end, they will have a whole project report!
- to write on your preparation paper all of the needed notions you have to introduce.
- to have a notebook in which you put down all the interactions you have with each (group of) student(s).

In order to begin a such pedagogy, you can introduce a part of you course with this pedagogy the first year; the second year, you can teach the half program and the so on.

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